Accretion Disks Around Class 0 Protostars:
The Case of VLA 1623

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arXiv:astro-ph/9608087v1 13 Aug 1996

ABSTRACT

Continuum emission at 220 and 355 GHz from the prototype Class 0 source VLA 1623 has been detected using the JCMT-CSO interferometer. Gaussian fits to the data place an upper limit of 70 AU on the half-width half-maximum radius of the emission, which implies an upper limit of ~ 175 AU for the cutoff radius of the circumstellar disk in the system. In the context of existing collapse models, this disk could be magnetically supported on the largest scales and have an age of $\sim 6 \times 10^4$ yr, consistent with previous suggestions that Class 0 sources are quite young. The innermost region of the disk within ~ 6 AU is likely to be in centrifugal support, which is likely large enough to provide a drive for the outflow according to current theoretical models. Alternatively, if 175 AU corresponds to the centrifugal radius of the disk, the age of the system is $\sim 2 \times 10^5$ yr, closer to age estimates for Class I sources.

Subject headings: accretion: accretion disks – circumstellar matter – stars: formation – stars: low-mass, brown dwarfs – stars: pre-main-sequence

1. Introduction

Submillimeter observations of deeply embedded star-forming regions have identified a small number of strong sources that have virtually no emission at wavelengths below 10 μm and whose spectral energy distributions are characterized by single blackbodies at $T \simeq 15 - 30$ K. These cold "Class 0" sources are good candidates for protostars whose circumstellar envelope masses exceed that of their central protostars (André, Ward-Thompson, & Barsony 1993, André & Montmerle 1994, André 1995), in which case their ages may be as low as 2×10^4 yrs (Barsony 1994). Sources in this class are also defined by the presence of energetic bipolar outflows.

There is now good evidence that a large fraction of young stellar objects have dense, dusty accretion disks (eg. Beckwith et al. 1990, Lada 1991). This is in accord with standard collapse models of star formation (eg. Terebey, Shu, & Cassen 1984), which predict that systems older than 10^5 years have centrifugally supported disks extending to 100 AU. Disks around the sources L1551 IRS5 and HL Tau have been resolved with semi-major axes of 80 and 60 AU, respectively (Lay et al. 1994, Mundy et al. 1996). Even larger magnetically supported "pseudo" disks extending out to 1000 AU are predicted for some magneto-hydrodynamic collapse models (Galli & Shu 1993, Fiedler & Mouschovias 1993).

Most current models of outflow posit the existence of disks as an essential element of the engine (eg. Pudritz, Pelletier, & Gomez de Castro 1991, Konigl & Ruden 1993, Shu et al. 1993). If the Class 0 sources are young, then the Terebey et al. (1984) models suggest that a disk with radius of 100 AU at 2×10^5 yrs has a centrifugal radius of only $r_c \propto t^3 \simeq 0.8$ AU at 2×10^4 yrs. Similarly a larger magnetically supported outer region of $r_B \propto t^{7/3} \simeq 20$ AU (Galli & Shu 1993) should be present at these early times if 1000 AU structures exist at 2×10^5 yrs. Such small, centrifugally supported regions would still be able to drive outflows, while the larger magnetic structure could play an important role

in star formation itself. This *Letter* describes the results of our search for an accretion disk in the prototype Class 0 source, VLA 1623, using the JCMT-CSO submillimeter interferometer.

2.   Observations and Analysis

Observations of VLA 1623 were obtained with the JCMT-CSO interferometer on 7 April 1995 at a frequency of 356 GHz with 500 MHz bandwidth and 9 April 1995 at 353 GHz with 1 GHz bandwidth. The projected length of the baseline ranged from 140 to 190 kλ and observations were obtained over a period of two to three hours each night. The quasar NRAO 530 was used to monitor the gain of the instrument and the absolute uncertainty in the flux calibration is 20%. The average gain of the instrument was 70 Jy K^{-1} and the system temperature was 800 K. The data were obtained with 10 s integrations and vector-averaged over 100 s intervals. Since the phase response of the interferometer over longer periods of time is not yet known, only the amplitude of the visibility function was used in the analysis. Observations were also obtained at a frequency of 220 GHz with 1 GHz bandwidth on 10 April 1995 over a period of 7 hours. The projected length of the baseline ranged from 60 to 120 kλ and the system temperature was 400 K.

The visibility flux of VLA 1623 shows no dependence on the projected baseline length (Figure 1a,b) and thus the source is unresolved. We measured both the flux and an upper limit to the full-width half-maximum radius of the source using the techniques outlined in Lay et al. (1994). Briefly, we fit a two-dimensional gaussian to the data using the Rice distribution to describe the noise. The position angle was fixed at 35° to orient the disk perpendicular to the large-scale outflow (André et al. 1990). Fifty simulated data sets were then generated by adding artificial noise to the best fitting model. The best fit parameters (flux and major-axis radius) derived from these simulated data sets are shown in Figure 1d,

from which 90% confidence limits can be determined. At 355 GHz, the flux is $0.5^{+0.4}_{-0.1}$ Jy and the major-axis radius is $R < 70$ AU (for a distance to ρ Oph of 160 pc). Fitting the data with a circularly symmetric gaussian produces a somewhat smaller upper limit to the radius of 45 AU. A more physically valid model would be a power-law disk (see §3). The large uncertainty in the flux at 355 GHz is due to the rather limited baseline coverage obtained for this low declination source. As a result, the model fits are unable to distinguish between a weaker compact source and a stronger, more extended source for which some of the flux is resolved out even on the shortest baselines at 355 GHz (Figure 1). At 220 GHz, the flux is $0.12^{+0.08}_{-0.02}$ Jy and the radius is $R < 150$ AU.

We calculated the expected contribution to the visibility flux from the extended envelope as in Lay et al. (1994). We modeled the envelope of VLA 1623 as an optically thin sphere with $T \propto r^{-0.5}$ and $\rho \propto r^{-1.5}$. The flux scale was calibrated using single dish fluxes from André et al. (1993). We also assumed that the envelope stops where the disk begins and so adopted an inner cutoff radius for the envelope of 70 AU. Under these conditions, the envelope for the 70 AU model contributes negligible flux at 355 GHz and only 0.03 Jy at 220 GHz. Thus, we attribute most of the observed flux to the presence of an unresolved disk in VLA 1623.

3. Fitting Accretion Disk Models

Our observations provide useful constraints on plausible accretion disk models for the origin of the submillimeter emission. We adopt the approach of Beckwith et al. (1990) and assume a geometrically thin disk with power law scalings with radius of the disk temperature and surface density, $T(r) = T_o(r/r_o)^{-q}$ and $\Sigma(r) = \Sigma_o(r/r_o)^{-p}$, where T_o and Σ_o are the dust temperature and mass surface density at the inner radius r_o of the dust distribution. The optical depth is $\tau = \Sigma \kappa_\nu / cos\theta$, where θ is the inclination of the disk axis

to the line of sight and $\kappa_\nu = \kappa_o(\nu/\nu_o)^\beta$ is the specific dust opacity. The flux emitted by the disk is then $F_\nu = (2\pi cos\theta/D^2) \int_{r_o}^{R_D} B_\nu(T)(1 - e^{-\tau})rdr$ where $B_\nu(T)$ is the Planck function, D is the distance to the source, and R_D is the outer disk cutoff radius. Power-law fits to disks around HL Tau and L1551 IRS5 produce cutoff radii up to a factor of 2.5 larger than radii from gaussian fits (Lay et al. 1994, Mundy et al. 1996). However, since VLA 1623 is unresolved, there is no structure in the visibility data to constrain a power law and so such a fit was not performed on this data set.

While $q = 0.75$ for purely viscous accretion disk models, more general calculations which incorporate the effects of reprocessing and disk flaring typically find $0.5 \leq q \leq 0.75$. Noting that various model fits have $0.5 \leq p \leq 1.5$ (eg. Thamm, Steinacker, & Henning, 1994), we adopt ranges of $0.25 \leq q \leq 0.75$ and $-0.25 \leq 2 - p - q \leq 1.25$. The maximum dust temperature must be less than its sublimation temperature, $T_{o,max} \sim 2000$ K. The inner edge of the disk cannot be inside the stellar photosphere, which for low mass stars gives $r_o \geq R_* \simeq 2$ R$_\odot = 0.01$ AU. We take the inclination angle of the disk to be that of the outflow from VLA 1623, $\theta = 80^o \pm 5^o$ or almost edge-on (André et al. 1990). The spectrum $F_\nu \propto \nu^\alpha$ determined from our measured fluxes at 355 and 220 GHz has $\alpha \simeq 3.0^{+1.6}_{-2.2}$, where optically thin models have $\alpha = 2 + \beta$ and optically thick models have $\alpha = 2$. Recent calculations show that $\beta \simeq 1.5 \pm 0.5$ at wavelengths from 650 μm to 2.7 mm for submicron to millimeter sized particles and a wide variety of densities and temperatures (Pollack et al. 1994). Finally, the parameter space characterizing the emission is also constrained by our observed flux, our upper limit to the cutoff radius of $\sim 2.5 \times 70 = 175$ AU, and by the ~ 1 L$_\odot$ bolometric luminosity of the entire source (André et al. 1993).

It is obvious that this accretion disk model covers a large parameter space that is relatively poorly constrained. The issue of the uniqueness of model fits has been addressed by Thamm et al. (1994) and Bouvier & Bertout (1992), who find "global ambiguity"

in acceptable fits to models of T Tauri disks based on spectral energy distributions that incorporate far more infrared data than is available for VLA1623. However, for the limiting cases of completely optically thick or thin truncated disks in the Rayleigh-Jeans limit, the cutoff radius can be related to the observed flux F_ν by

$$R_D/r_o = [(\gamma \bar{L}_\nu/x cos\theta) + 1]^{1/\gamma} \tag{1}$$

where $\bar{L}_\nu = D^2 F_\nu / 2\pi (2k\nu^2/c^2)$ can be computed from the observations. For the optically thick case, the parameters are $\gamma = 2 - q$ and $x = T_o r_o^2$, while for the optically thin case, $\gamma = 2 - p - q \neq 0$ and $x = \tau_o T_o r_o^2$, where $\tau_o \equiv \kappa_\nu \Sigma_o / cos\theta$.

Equation (1) shows that we can simplify our analysis of the model by reducing the six free parameters (κ_ν, q, p, T_o, r_o, Σ_o) to essentially the two parameters γ and x (with R_D measured in units of r_o). Then a disk model characterized by a given index γ produces a unique curve in the $R_D/r_o - x$ plane. These curves are plotted in Figure 2 for VLA 1623, for which $F_\nu = 0.5$ Jy gives $\bar{L}_{355} = 2.25 \times 10^4$ K AU^2. In addition, our observed flux and upper limit to the cutoff radius restrict possible disk models to lie within the region demarcated by the three dashed lines in Figure 2. Boundary line (A) is the $\gamma = 2$ model (i.e. $q = 0$ or $p + q = 0$ for optically thick or thin cases), which demarcates a model disk with uniform surface brightness and therefore the smallest cutoff radius R_D for the emitted flux. Line (B) is our upper limit to the cutoff radius $R_D \leq 175$ AU coupled with the condition that $r_o \geq 0.01$ AU, while line (C) is the upper limit to R_D/r_o for a given x. Line (C) arises from the fact that T_o must be less than $T_{o,max}$, so that r_o must be greater than $(x/T_{o,max})^{1/2}$, and thus $R_D \leq 175$ AU implies $R_D/r_o \leq 175(T_{o,max}/x)^{1/2}$. Figure 2 shows that while optically thick models with values of $q \simeq 0.25 - 0.5$ fill the largest portion of the permitted parameter space, models with q up to 0.75 are also acceptable. Optically thin models fill a much smaller portion of parameter space, with low values of $2 - p - q$ being less likely. In general, the optically thin solutions have relatively large values for the inner radius, which

suggests that they may also have a small optically thick region that contributes relatively little to the total flux of the source.

The observed bolometric luminosity of the envelope+disk system also provides a constraint on the cutoff radius. For $q \neq 0.5$, the luminosity of an optically thick disk is given by $L_{bol} = 4\pi\sigma T_o^4 r_o^2((R_D/r_o)^{2-4q} - 1)/(2 - 4q)$, while for $q = 0.5$, $L_{bol} = 4\pi\sigma T_o^4 r_o^2 ln(R_D/r_o)$. For $q < 0.5$, this expression can be combined with equation (1) to give

$$R_D = 47.7\,\text{AU}\frac{(2-q)^{2/3}}{(2-4q)^{1/6}}\Big(\frac{L_{bol}}{1\,\text{L}_\odot}\Big)^{-1/6}\Big(\frac{L_{355}/2.25 \times 10^4\,\text{K\,AU}^2}{cos\theta/cos80}\Big)^{2/3} \qquad (2)$$

In addition, equation (1) can be rewritten as $T_o r_o^q \propto R_D^{q-2}$, which allows the straightforward calculation of $T_o r_o^q$ for a given R_D obtained from equation (2). For $q = 0$ and $L_{bol} < 1\,\text{L}_\odot$, the cutoff radius is $R_D > 68$ AU and the inner disk temperature is $T_o < 57$ K, while for $q = 0.25$ and $r_o = 0.01 - 10$ AU, the radius is >69 AU and the temperature is $T_o < 430 - 80$ K. For $q \geq 0.5$ the expression for R_D is different from equation (2) and depends on the value of r_o. For $q = 0.5$ and $r_o = 0.01 - 1$ AU, the cutoff radius is >90-85 AU, while for $q = 0.75$ the radius is >230-120 AU. A smaller but warmer optically thin disk could probably also produce the observed 355 GHz flux and bolometric luminosity, but the numerical models required to derive the exact cutoff radius in this case are beyond the scope of this paper.

Thus both the spectral index and the models summarized in Figure 2 are consistent with either optically thin or optically thick disks with a cutoff radius not exceeding 175 AU. We obtain a lower limit to the disk mass from the measured 220 GHz flux by taking the optically thin limit,

$$M \geq (D^2 F_\nu/\kappa_\nu B_\nu(T_o))[2 - p - q/2 - p](R_D/r_o)^q \qquad (3)$$

Assuming a cutoff radius $R_D = 175$ AU, an inner radius $r_o = 0.01$ AU, an opacity $\kappa_\nu = 0.01$ cm^2 gm^{-1} at 220 GHz ($\beta = 1.5$), a maximum temperature of $T_{0,max} = 430$ K, and $q \simeq 0.25$ gives a minimum disk mass $M \geq 3 \times 10^{-2}$ M$_\odot$; the true disk mass could be much higher.

4. Discussion

The constraints on the size of the central source have implications for theoretical models of the evolution of sources such as VLA 1623. We adopt an effective sound speed $a = 0.27$ km s^{-1} for this innermost region of VLA 1623 (André et al. 1993) and a magnetic field strength $B = 30$ μG appropriate for low mass molecular cloud cores (Myers & Goodman 1988). In the context of the Galli & Shu (1993) model, the magnetic support radius of the disk is given by

$$r_B = 0.12(G^2 B^4/a)^{1/3} t^{7/3} = 2.75(B/30\,\mu\mathrm{G})^{4/3}(a/0.27\,\mathrm{km\ s^{-1}})^{-1/3} t_4^{7/3}\ \mathrm{AU} \qquad (4)$$

where t_4 is in units of 10^4 yrs and G is the gravitational constant. If we identify the upper limit to the cutoff radius of 175 AU with the radius of the pseudo-disk, r_B, then this model predicts an age for the system of $t < 5.9 \times 10^4 (B/30\,\mu\mathrm{G})^{-4/7}(a/0.27\,\mathrm{km\ s^{-1}})^{1/7}$ yr. This age agrees with general estimates for Class 0 ages (Barsony 1994), but is somewhat longer than initial estimates for the age of VLA 1623 (André et al. 1993). In a collapsing singular isothermal sphere (SIS) model, the expansion wave has reached a radius $r_{exp} = a t_{exp} < 3400(t_{exp}/5.9 \times 10^4\ \mathrm{yr})$ AU, which is comparable to the size of the 2000 AU envelope seen in the André et al. (1993) map. In the context of the SIS model, this age suggests that the envelope and the collapsed protostellar core+disk will each contain comparable amounts of material.

Alternatively, if we identify 175 AU with the centrifugal balance radius, $r_c = 0.058 a \Omega_o^2 t^3 = 0.030(\Omega_o/10^{-13} s^{-1})^2 t_4^3$ AU, then the age of the system is $< 1.8 \times 10^5$ yr, similar to age estimates for Class I sources (Wilking, Lada, & Young 1989). In this picture, the magnetic pseudo-disk, if present, would extend to 2300 AU, and must have a very low column density in order to be undetectable with lower resolution measurements (e.g. André et al. 1993). This age is five times longer than the time required for the expansion wave to reach the edge of the circumstellar envelope. Over this time, a compressional wave

propagating inwards from the envelope edge would have accelerated the collapse of the remaining envelope material onto the stellar core.

Returning to the assumption that most of the compact core is magnetically supported, the centrifugal balance radius in the pseudo-disk then occurs at $r_c < 6.2(\Omega_o/10^{-13}s^{-1})^2$ AU. This region is large enough either to drive off a disk wind or to interact with a possible stellar magnetosphere, and thus theoretical models developed to explain outflows in Class I sources are likely to be equally valid for Class 0 sources. The total collapsed mass of the star+disk system in the SIS model is $M_{coll} \simeq [1 + (\pi/4)]^{-1}2a^2r_{exp}/G = 0.29$ M$_\odot$. Taking a stellar radius $R_* = 2.5$ R$_\odot$ (valid for models with $M_* < 0.3$ M$_\odot$, Stahler 1988) and a bolometric luminosity $L_* \sim 1$ L$_\odot$ (André et al. 1993), we find that an accretion rate through the disk of $\dot{M}_a = L_*R_*/GM_* = 0.27 \times 10^{-6}$ M$_\odot$ yr^{-1} can provide the observed luminosity of VLA 1623. This accretion rate is significantly smaller than the rate inferred from the collapse of an SIS model, $\dot{M}_i = 0.975a^3/G = 4.5 \times 10^{-6}$ M$_\odot$ yr^{-1}. A discrepancy of a factor of 5 between the disk and collapse accretion rates in SIS models has been noted by Kenyon, Calvet, & Hartmann (1993) for more evolved T Tauri systems. A possible reason for the inconsistency in the mass accretion rates is that SIS models have constant infall rates. More general models of molecular cloud cores (eg. McLaughlin & Pudritz 1996, 1997, Foster & Chevalier 1993) do not collapse at a constant accretion rate, which may alleviate this problem.

Finally, we note that the outflow in VLA 1623 has reached much larger scales (~ 20000 AU, André et al. 1990) than the expansion wave predicted by the SIS model. This situation is possible because outflows are likely to be super-Alfvénic. The effect that such an outflow has upon collapse has yet to be investigated.

5.   Conclusions

High resolution submillimeter continuum observations have detected a compact source in VLA 1623, which is most readily interpreted as a disk. Gaussian fitting gives an upper limit of 70 AU for the half-width half-maximum radius, which implies an upper limit of ~ 175 AU for the cutoff radius of the circumstellar disk in the system. In the context of existing collapse models, such a structure could be magnetically supported on the largest scales and have an age $\sim 6 \times 10^4$ yr. The innermost region of the disk within ~ 6 AU is likely to be in centrifugal support, which could provide a drive for the outflow according to current theoretical models. Alternatively, if the entire observed disk is in centrifugal support, then the age of the system is $\sim 2 \times 10^5$ yr, similar to age estimates of Class I sources. In this case, we may be detecting a Class I source viewed nearly edge-on.

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Fig. 1.— (a) The visibility flux of VLA 1623 at 220 GHz detected with the JCMT-CSO interferometer as a function of the projected baseline length. The flux shows no dependence on projected baseline length and thus the source is unresolved. (b) The visibility flux at 355 GHz as a function of baseline length. (c) The visibility flux at 355 GHz as a function of the hour angle of the observations. (d) The source flux at 355 GHz deduced from 50 Monte Carlo simulations as a function of the deduced upper limit to the source size. With the available data it is impossible to distinguish between a weak compact source and a more extended, somewhat stronger source. The dashed line indicates the 90% confidence level upper limit to the radius of the disk.

Fig. 2.— (a) Cutoff radius R_D of an optically thick disk in units of the inner disk radius r_o versus the product $T_o r_o^2$, where T_o is the temperature of the disk at radius r_o. Solid lines are model fits for different values of q, the exponent in the temperature gradient. Viable accretion disk models lie within the region demarcated by the dashed line segments, A,B,C, described in the text. The angle between the disk axis and the line of sight is $\theta = 80^o$. (b) As in (a), but for optically thin emission. In this case R_D depends on the product $\tau_o T_o r_o^2$.

Figure showing four panels. Top left: Visibility Flux (Jy) versus Projected Baseline (kλ) labeled "220 GHz". Top right: Visibility Flux (Jy) versus Hour Angle labeled "355 GHz". Bottom left: Visibility Flux (Jy) versus Projected Baseline (kλ) labeled "355 GHz". Bottom right: Model Total Flux (Jy) versus Radius (AU) labeled "Fits to Simulated 355 GHz Data".

Left panel:

B

.25 .35 .5 .65 q=.75

A

C

Optically Thick
$\theta = 80°$
$T_{o,max} = 2000$ K

$\log (R_D / r_o)$

$\log (T_o r_o^2)$

Right panel:

B

1.25 1.00 .75 .50 .25

$2-p-q$
$= -.25$

A

C

Optically Thin
$\theta = 80°$
$T_{o,max} = 2000$ K

$\log (R_D / r_o)$

$\log (\tau_o T_o r_o^2)$